State University of New York College at Brockport

SPS Undergraduate Research Award Final Report

"Chaos in a Sinusoidally Driven Resistor-Inductor-Diode Circuit and in a Driven, Damped Torsion Pendulum"

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Introduction

Under this grant a driven torsion pendulum, which has the possibility of behaving chaotically was purchased. In addition, the department of physics at SUNY Brockport purchased state-of-the-art data acquisition hardware and software. This allows the simultaneous taking of two data streams from a Resistor-Inductor-Diode circuit which we know behaves chaotically. Previously, we had not been able to take data simultaneously. In what follows, we report on the results of the two experiments.

The Resistor-Inductor-Diode Circuit

In this experiment, we want to learn about the dynamic behavior of a diode driven at frequencies greater than its operating frequency. We feel that its chaotic behavior may help answer the question.

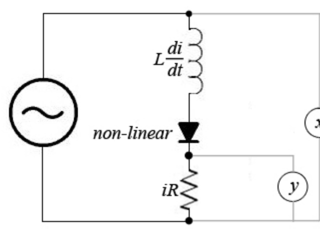

Figure 1

The circuit is constructed by connecting a resistor, inductor and diode in series with a sinusoidal voltage source, figure 1.

The differential equation governing the circuit is given by Kirchoff's Voltage rule:

$$V_R + V_L + V_D = V_0 \cos \omega t + V_{offset} \qquad \textbf{Equation 1}$$

Where:

V_R = voltage across the resistor = iR

V_L = voltage across the inductor = $L\frac{di}{dt}$

V_D = voltage across the diode, whose analytical form is unknown at this time.

V_0 = the amplitude of the driver

ω = the frequency of the driver

$$iR + L\frac{di}{dt} + V_D = V_0 \cos \omega t + V_{offset}$$ **Equation 2**

We have found this circuit to exhibit both periodic and chaotic behavior.

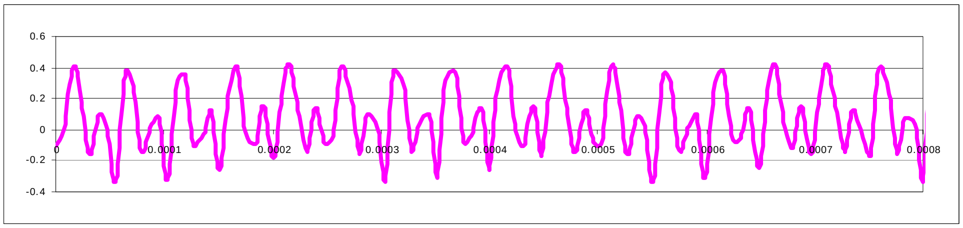

Figure 2

Figure 2 is a plot of V_R vs. time for "period-two" behavior. This means that the voltage across the resistor goes through one cycle during the time that it takes the source voltage to go through two cycles. The periodicity is not evident in figure 2, however it is if a Lissajous figure, a plot of V_L vs. V_R, is constructed. This is shown in figure 3, the two loops in this plot correspond to period-two behavior.

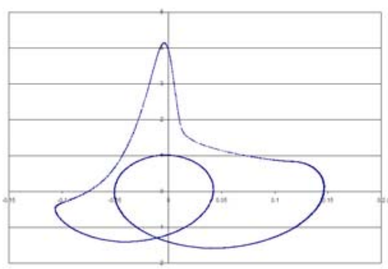

Figure 3

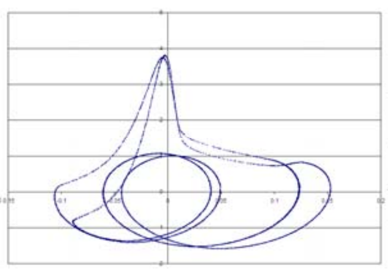

Figure 4

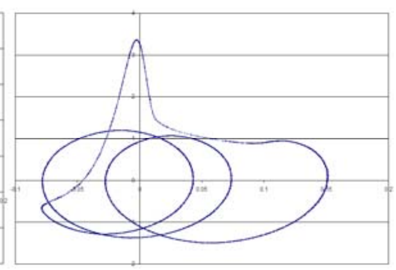

Figure 5

Figures 4 and 5 correspond to periods four and three, respectively.

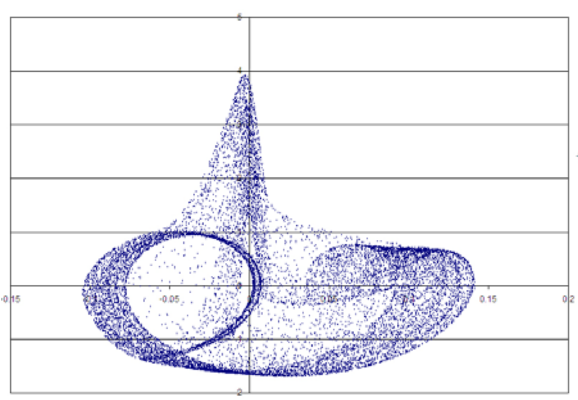

Figure 6

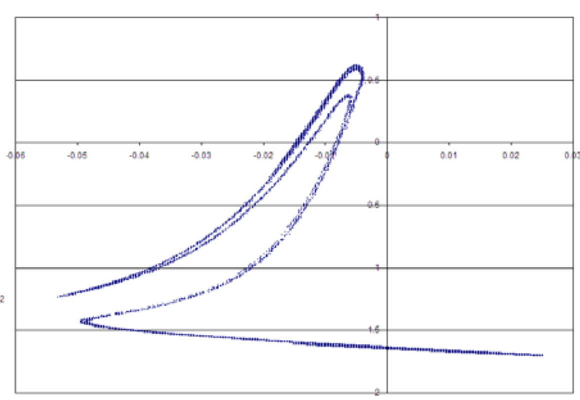

Figure 7

Figure 6 corresponds chaotic behavior. All of the data was taken at about 40 kilohertz by varying the DC offset of the driver only. All of these Lissajous figures are referred to as phase-space plots.

Figure 7 is a Poincaré section of figure 6 and has the "textbook" shape of a strange attractor. Strange attractors vary in shape and style but, in general, belong to a class of geometrical objects called fractals, objects that have a non-integer dimension. This places them on the dimensional scale somewhere between a point and a line, between a line and a surface or a surface and a volume.

The dimension of the attractor was measured using the correlation method. This method consists of selecting a representative sample of points distributed throughout the attractor. For each point, a circle of a small radius, ε, is drawn. The number of points that fall within the circle for each of the selected points is counted and the average calculated. The value of ε is increased until every sample point is closer than ε away from every other point.

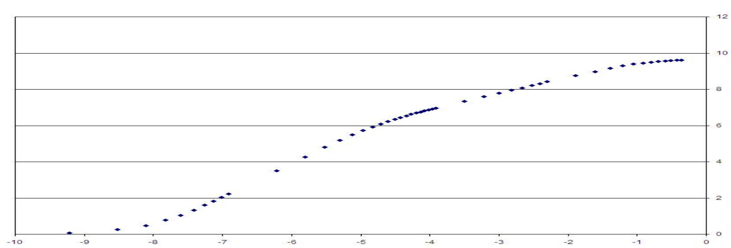

Figure 8

The dimension is obtained by plotting the natural log of the radius of the circles around each sample point versus the natural log of the average number of points that fall within the corresponding circle (figure 8). The dimension is the slope of the linear portion of this curve. In this case, the slope of the linear region is approximately 1.1. We need to interpret what this number is telling us.

The Torsion Pendulum

In this experiment, we want to get a feel for chaos, by "seeing" what chaos looks like in a mechanical system. The feel we obtain should give us a conceptual insight to the mathematical analysis of the RLD circuit.

We, indeed, have observed long-term behavior which appears to be chaotic, however, "long-term" has been on the order of hours only; eventually, all runs end up with the pendulum stuck in a potential well.

The apparatus came with two sets of weights which, we feel, are too light and too heavy, respectively.

We have constructed four additional sets of intermediate weights in our machine shop. Another student, doing research in the low-temperature laboratory, did the machining. We will be testing these new weights in the spring 2006 semester. We did learn about the sensitivity of behavior of non-linear systems when setting up to run the pendulum. Its behavior is especially sensitive to the driving frequency.

Summary

The new data acquisition hardware and software has given us a sense that we are on the right track in determining the dimensionally of the RLD circuit. The torsion pendulum has been difficult to drive to chaos, we anticipate using the new set of weights will remedy that.

Budget

Through the grant, the following items were purchased:

Leybold Torsion Pendulum	$	1,686.00
Torsion Pendulum Power Supply		594.00
Total	$	2,280.00
Less grant amount		-2,000.00
Balance	$	280.00

The balance remaining from those items plus some additional items were purchased by the SUNY Brockport Department of Physics.

Balance	$	280.00
National Instruments LabVIEW		site license
National Instruments Data Acquisition Board NI PCI-6143		995.00
National Instruments Terminal Block NI BNC-2110		295.00
Twisted Pair Cable with Shielding NI SHC68-68-EP		95.00
Total SUNY Brockport Contribution	$	1,665.00

Acknowledgements

We would like to thank Sigma Pi Sigma for the award of the 2004-2005 Undergraduate Research Grant, the State University of New York, College at Brockport for their continued financial support and use of their facilities to conduct this research, the SUNY Brockport Chapter of SPS for their dedication to the support of undergraduate research and Professors Richard V. Mancuso and Mohammed Z. Tahar for their insight and support of the students conducting research in this project and others within the Department of Physics.