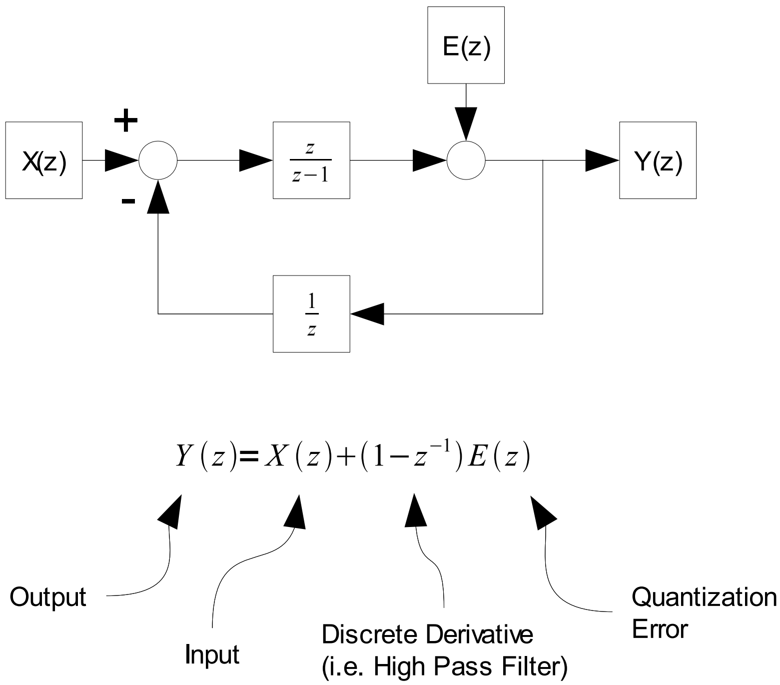
Noise Shaping in MASH Delta-Sigma Converters

Joey White

Hephaestus Audio

Standard Delta-Sigma Converter

Delta-sigma modulators move noise out-of-band. The total noise is not decreased, however it is shifted above the bandwidth of interest. Higher-order delta-sigma modulators have a more abrupt transition than lower-order modulators, similar to the more abrupt rolloff of a higher-order electronic filter.



$$Y(z) = X(z) + (1 - z^{-1}) E(z)$$

Figure 1 - 1st order delta-sigma converter noise model

Fig. 1 gives the transfer function for a simple 1st order delta-sigma converter. For higher order converters, the general quantization error transfer function is:

$$Y(z) = X(z) + (1 - z^{-1})^N E(z)$$

Where N is the order of the converter (i.e. the number of stages). Thus the noise shaping function is a differentiator (i.e. high pass filter) with order equal to the order of the converter.

Higher-order delta-sigma structures may be realized, however 3rd order converter and above are unstable unless special measures are taken. This is due to the global feedback plus the phase shift of the individual stages.

MASH Delta-Sigma Converter

An alternative structure, that avoids instability issues by cascading lower-order sections, is the multi-stage noise shaping (MASH) delta-sigma converter.

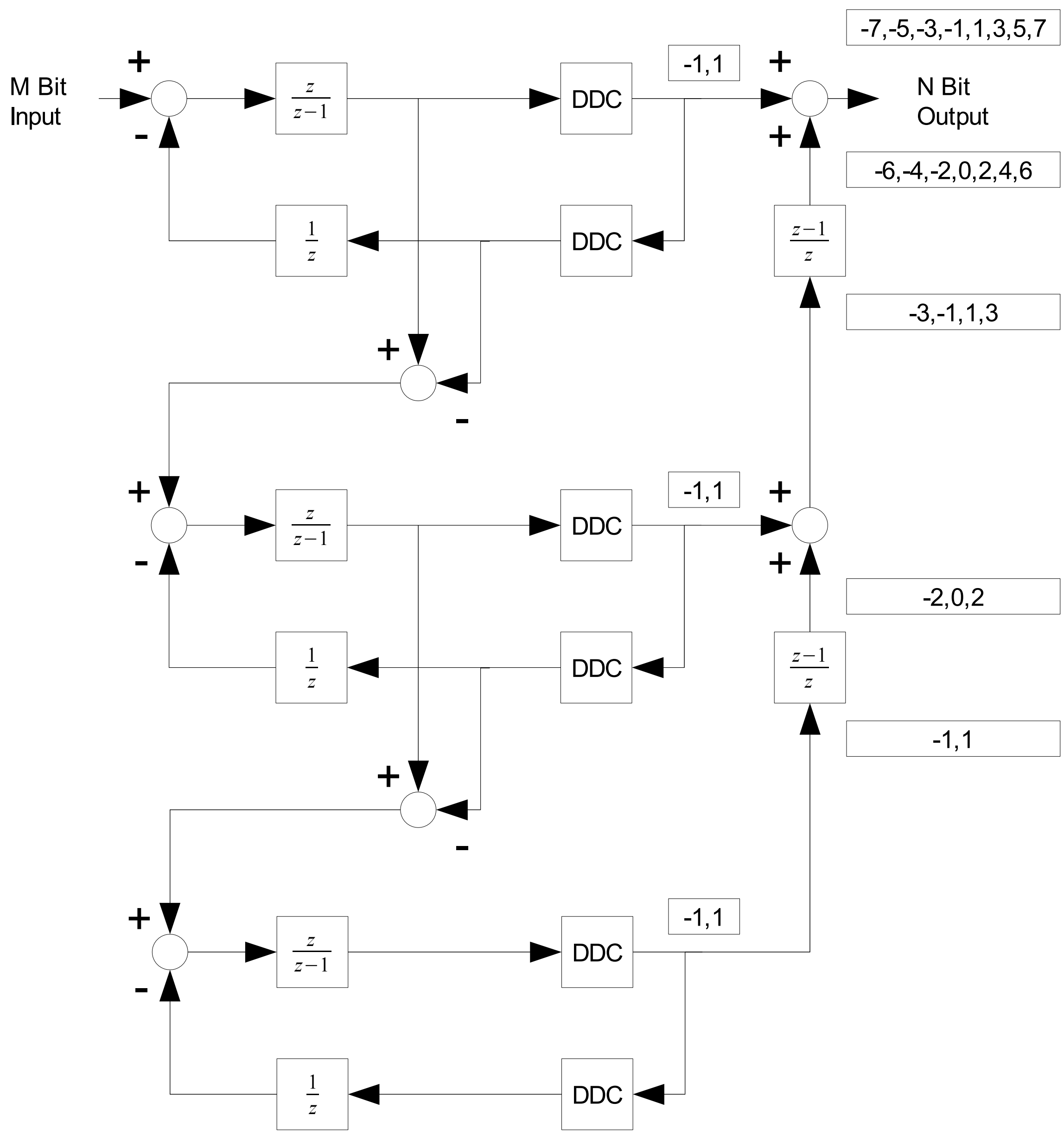


Figure 2 - 3rd order delta-sigma MASH converter

MASH delta-sigma converters work by canceling the quantization error of each preceding stage. What is leftover at the output is the quantization error of the final delta-sigma stage passed through a discrete differentiator chain. It is important to note however that the output of a MASH converter is not a bitstream output, but a PCM output with 2^N discrete levels, where N is the order of the MASH converter (i.e. the number of stages).

MASH Delta-Sigma Transfer Function

Each stage has the transfer function of a 1st order delta-sigma converter. The quantization error of each stage is then fed to the following stage, as is shown in Fig. 2 for a 3rd order MASH converter.

1st Stage

With input X the transfer function of the 1st stage is:

$$Y_1 = X + (1+z^{-1})Q_1$$

Where Q_1 is the quantization error of the 1st stage.

2nd Stage

The quantization error of the 1st stage Q_1 is the input of the 2nd stage.

$$Y_2 = -Q_1 + (1+z^{-1})Q_2$$

Where Q_2 is the quantization error of the 2nd stage.

3rd Stage

The quantization error of the 2nd stage Q_2 is the input of the 3rd stage.

$$Y_3 = -Q_2 + (1+z^{-1})Q_3$$

Where Q_3 is the quantization error of the 3rd stage.

Net Transfer Function

Taking the output of each stage and summing them via the differentiator chain:

$$Y = Y_1 + (1+z^{-1})Y_2 + (1+z^{-1})^2 Y_3$$

$$Y = X + (1+z^{-1})Q_1 + (1+z^{-1})(-Q_1 + (1+z^{-1})Q_2) + (1+z^{-1})^2(-Q_2 + (1+z^{-1})Q_3)$$

$$Y = X + (1+z^{-1})^3 Q_3$$

This is the same transfer function as a standard 3rd order delta-sigma converter, however this structure is stable! Again, remember that the output is now multi-bit, instead of single-bit. Also note that the cancellation of quantization error requires *precise* matching of the individual stages. This is automatic for a digital-to-digital converter (DDC), but may be challenging for an analog-to-digital converter (ADC).

MASH Delta-Sigma Noise Floor

Translating the MASH delta-sigma transfer function to the noise floor across the audio bandwidth, we see it is possible to push the "knee" of the rise in quantization error just beyond the upper end of the audio bandwidth, with higher-order delta-sigma converters.

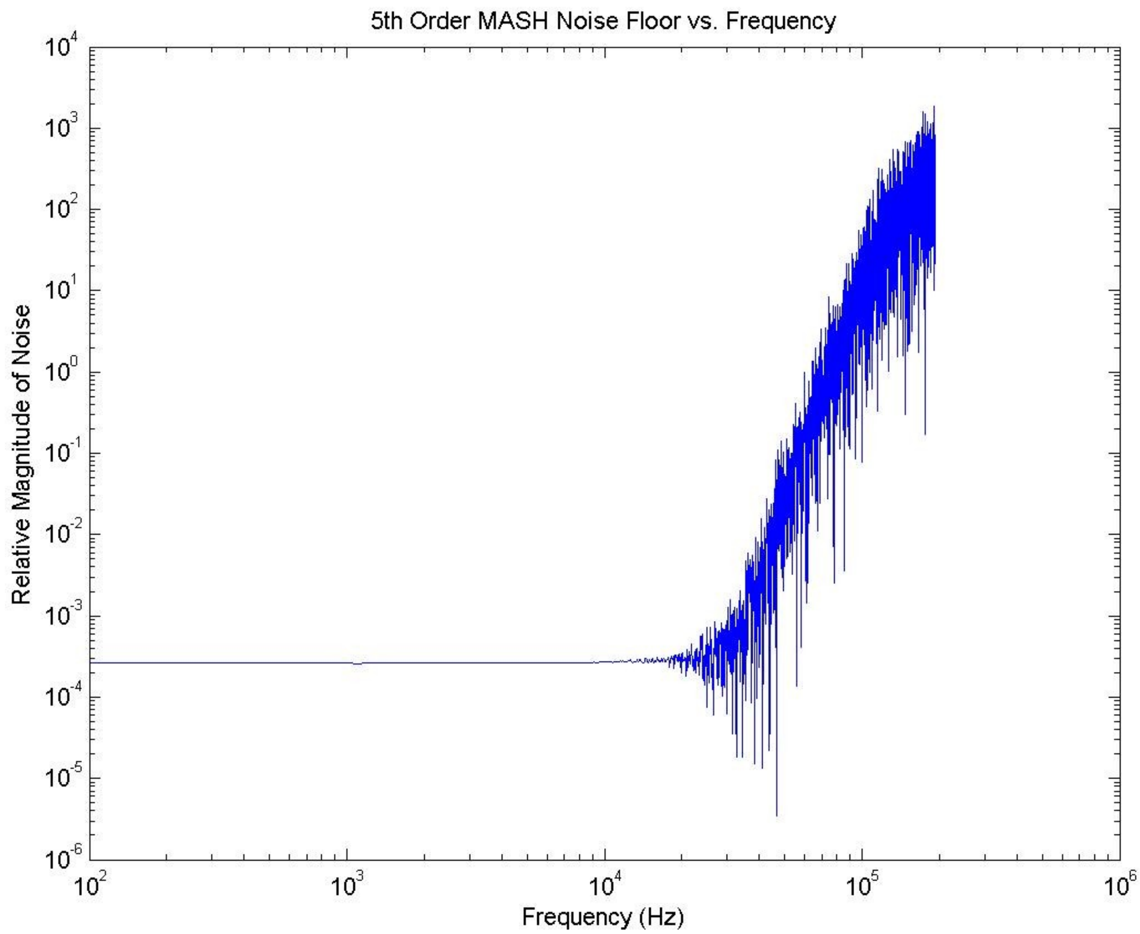


Figure 3 – Noise floor for a 5th order MASH converter

Fig. 3 shows how the bulk of the quantization error is pushed out just beyond 20kHz (the upper end of the audio bandwidth). Note how the rise in quantization error after this point is very abrupt (i.e. a 5th order function).

In summary, the MASH delta-sigma converter is an excellent alternative to a standard delta-sigma converter if the following constraints are acceptable:

- The implementation is a DDC or an ADC with well-matched analog components
- It is acceptable to have a multi-bit output, rather than a single-bit output